Tennessee Math Standards

Summer 2017

Knox County Schools
Mathematics
3-5

Tennessee Math Standards

Introduction

The Process

The Tennessee State Math Standards were reviewed and developed by Tennessee teachers for Tennessee schools. The rigorous process used to arrive at the standards in this document began with a public review of the then-current standards. After receiving 130,000+ reviews and 20,000+ comments, a committee composed of Tennessee educators spanning elementary through higher education reviewed each standard. The committee scrutinized and debated each standard using public feedback and the collective expertise of the group. The committee kept some standards as written, changed or added imbedded examples, clarified the wording of some standards, moved some standards to different grades, and wrote new standards that needed to be included for coherence and rigor. From here the standards went before the appointed Standards Review Committee to make further recommendations before being presented to the Tennessee Board of Education for final adoption.

The result is Tennessee Math Standards for Tennessee Students by Tennesseans.

Mathematically Prepared

Tennessee students have various mathematical needs that their K-12 education should address.

All students should be able to recall and use their math education when the need arises. That is, a student should know certain math facts and concepts such as the multiplication table, how to add, subtract, multiply, and divide basic numbers, how to work with simple fractions and percentages, etc. There is a level of procedural fluency that a student's K-12 math education should provide him or her along with conceptual understanding so that this can be recalled and used throughout his or her life. Students also need to be able to reason mathematically. This includes problem solving skills in work and non-work related settings and the ability to critically evaluate the reasoning of others.

A student's K-12 math education should also prepare him or her to be free to pursue post-secondary education opportunities. Students should be able to pursue whatever career choice, and its post-secondary education requirements, that they desire. To this end, the K-12 math standards lay the foundation that allows any student to continue further in college, technical school, or with any other post-secondary educational needs.

A college and career ready math class is one that addresses all of the needs listed above. The standards' role is to define what our students should know, understand, and be able to do mathematically so as to fulfill these needs. To that end, the standards address conceptual understanding, procedural fluency, and application.

Conceptual Understanding, Procedural Fluency, and Application

In order for our students to be mathematically proficient, the standards focus on a balanced development of conceptual understanding, procedural fluency, and application. Through this balance, students gain understanding and critical thinking skills that are necessary to be truly college and career ready.

Conceptual understanding refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.

Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly. One cannot stop with memorization of facts and procedures alone. It is about recognizing when one strategy or procedure is more appropriate to apply than another. Students need opportunities to justify both informal strategies and commonly used procedures through distributed practice. Procedural fluency includes computational fluency with the four arithmetic operations. In the early grades, students are expected to develop fluency with whole numbers in addition, subtraction, multiplication, and division. Therefore, computational fluency expectations are addressed throughout the standards. Procedural fluency extends students' computational fluency and applies in all strands of mathematics. It builds from initial exploration and discussion of number concepts to using informal strategies and the properties of operations to develop general methods for solving problems (NCTM, 2014).

Application provides a valuable context for learning and the opportunity to practice skills in a relevant and a meaningful way. As early as Kindergarten, students are solving simple "word problems" with meaningful contexts. In fact, it is in solving word problems that students are building a repertoire of procedures for computation. They learn to select an efficient strategy and determine whether the solution(s) makes sense. Problem solving provides an important context in which students learn about numbers and other mathematical topics by reasoning and developing critical thinking skills (Adding It Up, 2001).

Progressions

The standards for each grade are not written to be nor are they to be considered as an island in and of themselves. There is a flow, or progression, from one grade to the next, all the way through to the high school standards. There are four main progressions that are composed of mathematical domains/conceptual categories (see the Structure section below and color chart on the following page).

The progressions are grouped as follows:

Grade	Domain/Conceptual Category
K	Counting and Cardinality
K-5	Number and Operations in Base Ten
3-5	Number and Operations – Fractions
6-7	Ratios and Proportional Relationships
6-8	The Number System
9-12	Number and Quantity
K-5	Operations and Algebraic Thinking
6-8	Expressions and Equations
8	Functions
9-12	Algebra and Functions
K-12	Geometry
K-5	Measurement and Data
6-12	Statistics and Probability

<table>
<tr><th>Kindergarten</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td>Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="2">Ratios and Proportional Relationships</td><td></td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations - Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="6">Geometry</td><td colspan="3">Geometry</td><td>Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics and Probability</td></tr>
</table>

Each of the progressions begins in Kindergarten, with a constant movement toward the high school standards as a student advances through the grades. This is very important to guarantee a steady, age appropriate progression which allows the student and teacher alike to see the overall coherence of and connections among the mathematical topics. It also ensures that gaps are not created in the mathematical education of our students.

Structure of the Standards

Most of the structure of the previous state standards has been maintained. This structure is logical and informative as well as easy to follow. An added benefit is that most Tennessee teachers are already familiar with it.

The structure includes:

- **Content Standards -** Statements of what a student should know, understand, and be able to do.

- **Clusters** - Groups of related standards. Cluster headings may be considered as the big idea(s) that the group of standards they represent are addressing. They are therefore useful as a quick summary of the progression of ideas that the standards in a domain are covering and can help teachers to determine the focus of the standards they are teaching.

- **Domains** - A large category of mathematics that the clusters and their respective content standards delineate and address. For example, Number and Operations – Fractions is a domain under which there are a number of clusters (the big ideas that will be addressed) along with their respective content standards, which give the specifics of what the student should know, understand, and be able to do when working with fractions.

- **Conceptual Categories** – The content standards, clusters, and domains in the 9th-12th grades are further organized under conceptual categories. These are very broad categories of mathematical thought and lend themselves to the organization of high school course work. For example, Algebra is a conceptual category in the high school standards under which are domains such as Seeing Structure in Expressions, Creating Equations, Arithmetic with Polynomials and Rational Expressions, etc.

Standards and Curriculum

It should be noted that the standards are what students should know, understand, and be able to do; but, they do not dictate how a teacher is to teach them. In other words, the standards do not dictate curriculum. For example, students are to understand and be able to add, subtract, multiply, and divide fractions according to the standards. Although within the standards algorithms are mentioned and examples are given for clarification, how to approach these concepts and the order in which the standards are taught within a grade or course are all decisions determined by the local district, school, and teachers.

Tennessee State Math Standards

The Standards for Mathematical Practice

Being successful in mathematics requires that development of approaches, practices, and habits of mind be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop within their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics.

Processes and proficiencies are two words that address the purpose and intent of the practice standards. Process is used to indicate a particular course of action intended to achieve a result, and this ties to the process standards from NCTM that pertain to problem solving, reasoning and proof, communication, representation, and connections. Proficiencies pertain to being skilled in the command of fundamentals derived from practice and familiarity. Mathematically, this addresses concepts such as adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive dispositions toward the work at hand. The practice standards are written to address the needs of the student with respect to being successful in mathematics.

These standards are most readily developed in the solving of high-level mathematical tasks. High-level tasks demand a greater level of cognitive effort to solve than routine practice problems do. Such tasks require one to make sense of the problem and work at solving it. Often a student must reason abstractly and quantitatively as he or she constructs an approach. The student must be able to argue his or her point as well as critique the reasoning of others with respect to the task. These tasks are rich enough to support various entry points for finding solutions. To develop the processes and proficiencies addressed in the practice standards, students must be engaged in rich, high-level mathematical tasks that support the approaches, practices, and habits of mind which are called for within these standards.

The following are the eight standards for mathematical practice:

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

A full description of each of these standards follows.

MP1: Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2: Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MP3: Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP4: Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5: Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a compass, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6: Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

MP7: Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 - 3(x - y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8: Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), (x - 1)(x2 + x + 1), and (x - 1)(x3 + x2+ x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Literacy Skills for Mathematical Proficiency

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others and analyze and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies.
2. Understand and use correct mathematical vocabulary.
3. Discuss and articulate mathematical ideas.
4. Write mathematical arguments.

Reading

Reading in mathematics is different from reading literature. Mathematics contains expository text along with precise definitions, theorems, examples, graphs, tables, charts, diagrams, and exercises.
Students are expected to recognize multiple representations of information, use mathematics in context, and draw conclusions from the information presented. In the early grades, non-readers and struggling readers benefit from the use of multiple representations and contexts to develop mathematical connections, processes, and procedures. As students' literacy skills progress, their skills in mathematics develop so that by high school, students are using multiple reading strategies, analyzing context-based problems to develop understanding and comprehension, interpreting and using multiple representations, and fully engaging with mathematics textbooks and other mathematics-based materials. These skills support Mathematical Practices 1 and 2.

Vocabulary

Understanding and using mathematical vocabulary correctly is essential to mathematical proficiency. Mathematically proficient students use precise mathematical vocabulary to express ideas. In all grades, separating mathematical vocabulary from everyday use of words is important for developing an understanding of mathematical concepts. For example, a "table" in everyday use means a piece of furniture, while in mathematics, a "table" is a way of organizing and presenting information. Mathematically proficient students are able to parse a mathematical term, definition, or theorem, provide examples and counterexamples, and use precise mathematical vocabulary in reading, speaking, and writing arguments and explanations. These skills support Mathematical Practice 6.

Speaking and Listening

Mathematically proficient students can listen critically, discuss, and articulate their mathematical ideas clearly to others. As students' mathematical abilities mature, they move from communicating through reiterating others' ideas to paraphrasing, summarizing, and drawing their own conclusions. mathematically proficient student uses appropriate mathematics vocabulary in verbal discussions, listens to mathematical arguments, and dissects an argument to recognize flaws or determine validity. These skills support Mathematical Practice 3.

Writing

Mathematically proficient students write mathematical arguments to support and refute conclusions and cite evidence for these conclusions. Throughout all grades, students write reflectively to compare and contrast problem-solving approaches, evaluate mathematical processes, and analyze their thinking and decision-making processes to improve their mathematical strategies. These skills support Mathematical Practices 2, 3, and 4.

Mathematics | Grade 2

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 2nd grade.

Operations & Algebraic Thinking
Students solve one- and two-step addition and subtraction contextual problems within 100 with an unknown in any position. Students should solve a variety of problem types in order to make connections among contexts, equations, and strategies (See Table 1 - Addition and Subtraction Situations). Students also represent these problems with objects, drawings, and/or equations. Students build upon previously taught strategies to mentally add and subtract within 30. Students know from memory all sums of two one-digit numbers and related subtraction facts.

Numbers & Operations in Base Ten
Students extend their understanding of the base-ten place value system to 1,000. This includes counting by ones, fives, tens, and hundreds. Students write numbers using standard form, word form, and expanded form. They deepen their understanding of different ways a number can be composed and decomposed. Students extend their understanding of place value, properties of operations, and the relationship between addition and subtraction to add and subtract within 1,000 and fluently add and subtract within 100 (See Table 3 - Properties of Operations). They add up to four two-digit numbers. They should also be able to explain why these strategies work. Students mentally add and subtract 10 or 100 from a given number 100-900.

Measurement & Data
In previous grades, students measured with non-standard units. Students in 2nd grade measure with standard units (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length. Students use addition and subtraction to solve contextual problems involving lengths in the same units and represent lengths on a number line.

Geometry

Students describe and analyze shapes by examining their sides and angles. Students recognize and draw shapes based on given attributes, such as draw a shape with 3 vertices. Students also are able to partition circles and rectangles into two, three, and four equal shares and rectangles into rows and columns, laying the foundation for fractions and area

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Represent and solve problems involving addition and subtraction. (See Table 1 - Addition and Subtraction Situations)	**2.OA.A.1** Add and subtract within 100 to solve one- and two-step contextual problems, with unknowns in all positions, involving situations of *add to*, *take from*, *put together/take apart*, and *compare*. Use objects, drawings, and equations with a symbol for the unknown number to represent the problem.
B. Add and subtract within 30.	**2.OA.B.2** Fluently add and subtract within 30 using mental strategies. By the end of 2nd grade, know from memory all sums of two one-digit numbers and related subtraction facts.
C. Work with equal groups of objects to gain foundations for multiplication.	**2.OA.C.3** Determine whether a group of objects (up to 20) has an odd or even number of members by pairing objects or counting them by 2s. Write an equation to express an even number as a sum of two equal addends. **2.OA.C.4** Use repeated addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Understand place value.	**2.NBT.A.1** Know that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 can be represented in multiple ways as 7 hundreds, 0 tens, and 6 ones; 706 ones; or 70 tens and 6 ones). **2.NBT.A.2** Count within 1000. Skip-count within 1000 by 5s, 10s, and 100s, starting from any number in its skip counting sequence. **2.NBT.A.3** Read and write numbers to 1000 using standard form, word form, and expanded form. **2.NBT.A.4** Compare two three-digit numbers based on the meanings of the digits in each place and use the symbols >, =, and < to show the relationship.
B. Use place value understanding and properties of operations to add and subtract. (See Table 3 - Properties of Operations)	**2.NBT.B.5** Fluently add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. **2.NBT.B.6** Add up to four two-digit numbers using properties of operations and strategies based on place value.

Cluster Headings	Content Standards
B. Use place value understanding and properties of operations to add and subtract. (See Table 3 - Properties of Operations)	**2.NBT.B.7** Add and subtract within 1000 using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction to explain the reasoning used. **2.NBT.B.8** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100– 900. **2.NBT.B.9** Explain why addition and subtraction strategies work using properties of operations and place value. (Explanations may include words, drawing, or objects.)

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Measure and estimate lengths in standard units.	**2.MD.A.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.MD.A.2** Measure the length of an object using two different units of measure and describe how the two measurements relate to the size of the unit chosen. **2.MD.A.3** Estimate lengths using units of inches, feet, yards, centimeters, and meters. **2.MD.A.4** Measure to determine how much longer one object is than another and express the difference in terms of a standard unit of length.
B. Relate addition and subtraction to length.	**2.MD.B.5** Add and subtract within 100 to solve contextual problems involving lengths that are given in the same units by using drawings and equations with a symbol for the unknown to represent the problem. **2.MD.B.6** Represent whole numbers as lengths from 0 on a number line and know that the points corresponding to the numbers on the number line are equally spaced. Use a number line to represent whole number sums and differences of lengths within 100.

C. Work with time and money.	**2.MD.C.7** Tell and write time in quarter hours and to the nearest five minutes (in a.m. and p.m.) using analog and digital clocks. **2.MD.C.8** Solve contextual problems involving dollar bills, quarters, dimes, nickels, and pennies using ¢ and $ symbols appropriately.

D. Represent and interpret data.	**2.MD.D.9** Generate measurement data by measuring lengths of several objects to the nearest whole unit. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **2.MD.D.10** Draw a pictograph and a bar graph (with intervals of one) to represent a data set with up to four categories. Solve addition and subtraction problems related to the data in a graph.

Geometry (G)

Cluster Headings	Content Standards
A. Reason about shapes and their attributes.	**2.G.A.1** Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Draw two-dimensional shapes having specified attributes (as determined directly or visually, not by measuring), such as a given number of angles or a given number of sides of equal length. **2.G.A.2** Partition a rectangle into rows and columns of same-sized squares and find the total number of squares. **2.G.A.3** Partition circles and rectangles into two, three, and four equal shares, describe the shares using the words *halves, thirds, fourths, half of, a third of, and a fourth of*, and describe the whole as *two halves, three thirds, four fourths*. Recognize that equal shares of identical wholes need not have the same shape.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Mathematics | Grade 3

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 3rd grade.

Operations and Algebraic Thinking

Students build on their understanding of addition and subtraction to develop an understanding of the meanings of multiplication and division of whole numbers. Students use increasingly sophisticated strategies based on properties of operations to fluently solve multiplication and division problems within 100 (See Table 3 - Properties of Operations). Students interpret multiplication as finding an unknown product in situations involving equal-sized groups, arrays, area and measurement models, and division as finding an unknown factor in situations involving the unknown number of groups or the unknown group size. Students use these interpretations to represent and solve contextual problems with unknowns in all positions. By the end of 3rd grade, students should know from memory all products of single-digit numbers and the related division facts.

Students use all four operations to solve two-step word problems and use place value, mental computation, and estimation strategies to assess the reasonableness of solutions. They build number sense by investigating numerical representations, such as addition or multiplication tables for the purpose of identifying arithmetic patterns. Students should solve a variety of problem types in order to make connections among contexts, equations, and strategies (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations).

Number and Operations in Base Ten

Students begin to develop an understanding of rounding whole numbers to the nearest ten or hundred. Students fluently add and subtract within 1000 using strategies and algorithms. Students multiply one-digit whole numbers by multiples of 10.

Number and Operations in Fractions

This domain builds on the previous skill of partitioning shapes in geometry. This is the first time students are introduced to unit fractions. Students understand that fractions are composed of unit fractions and they use visual fraction models to represent parts of a whole. Students build on their understanding of number lines to represent fractions as locations and lengths on a number line. Students use fractions to represent numbers equal to, less than, and greater than 1 and are able to generate simple equivalent fractions by using drawings and/or reasoning about fractions. Students understand that the size of a fractional part is relative to the size of the whole.

Measurement and Data

In 2nd grade, students tell time in five minute increments, measure lengths, and create bar graphs, pictographs, and line plots with whole number units. In 3rd grade, students tell and write time to the nearest minute and solve contextual problems involving addition and subtraction. They use appropriate tools to measure and estimate liquid volume and mass. Students draw scaled pictographs and bar graphs and answer two-step questions about these graphs. Students generate measurement data and represent the data on line plots marked with whole number, half, or quarter units. Students recognize area as an attribute of two-dimensional shapes and measure the area of a shape using the standard unit (a square) by finding the total number of same-sized units required to cover the shape without gaps or overlaps. Students connect area to multiplication and use multiplication to justify the area of a rectangle by decomposing rectangles into rectangular arrays of squares.

Geometry

Students understand that shapes in given categories have shared attributes and they identify polygons. Students continue their understanding of shapes and fractions by partitioning shapes into parts with equal areas and identify the parts with unit fractions.

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Represent and solve problems involving multiplication and division.	**3.OA.A.1** Interpret the factors and products in whole number multiplication equations (e.g., 4 x 7 is 4 groups of 7 objects with a total of 28 objects or 4 strings measuring 7 inches each with a total of 28 inches.) **3.OA.A.2** Interpret the dividend, divisor, and quotient in whole number division equations (e.g., 28 ÷ 7 can be interpreted as 28 objects divided into 7 equal groups with 4 objects in each group or 28 objects divided so there are 7 objects in each of the 4 equal groups). **3.OA.A.3** Multiply and divide within 100 to solve contextual problems, with unknowns in all positions, in situations involving equal groups, arrays, and measurement quantities using strategies based on place value, the properties of operations, and the relationship between multiplication and division (e.g., contexts including computations such as 3 x ? = 24, 6 x 16 = ?, ? ÷ 8 = 3, or 96 ÷ 6 = ?) (See Table 2 - Multiplication and Division Situations). **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers within 100. *For example, determine the unknown number that makes the equation true in each of the equations: 8 x ? = 48, 5 = ? ÷ 3, 6 x 6 =?*
B. Understand properties of multiplication and the relationship between multiplication and division. (See Table 3 - Properties of Operations)	**3.OA.B.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known (Commutative property of multiplication). 3 x 5 x 2 can be solved by (3 x 5) x 2 or 3 x (5 x 2) (Associative property of multiplication). One way to find 8 x 7 is by using 8 x (5 + 2) = (8 x 5) + (8 x 2). By knowing that 8 x 5 = 40 and 8 x 2 = 16, then 8 x 7 = 40 + 16 = 56 (Distributive property of multiplication over addition).* **3.OA.B.6** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
C. Multiply and divide within 100.	**3.OA.C.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of 3rd grade, know from memory all products of two one-digit numbers and related division facts.

Cluster Headings	Content Standards
D. Solve problems involving the four operations and identify and explain patterns in arithmetic.	**3.OA.D.8** Solve two-step contextual problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations). **3.OA.D.9** Identify arithmetic patterns (including patterns in the addition and multiplication tables) and explain them using properties of operations. *For example, analyze patterns in the multiplication table and observe that 4 times a number is always even (because 4 x 6 = (2 x 2) x 6 = 2 x (2 x 6), which uses the associative property of multiplication)* (See Table 3 - Properties of Operations).

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Use place value understanding and properties of operations to perform multi-digit arithmetic.	**3.NBT.A.1** Round whole numbers to the nearest 10 or 100 using understanding of place value. **3.NBT.A.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **3.NBT.A.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 *(e.g., 9 x 80, 5 x 60)* using strategies based on place value and properties of operations.

Number and Operations - Fractions (NF)

Limit denominators of fractions to 2, 3, 4, 6, and 8.

Cluster Headings	Content Standards
A. Develop understanding of fractions as numbers.	**3.NF.A.1** Understand a fraction, $\frac{1}{b}$, as the quantity formed by 1 part when a whole is partitioned into b equal parts (unit fraction); understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$. *For example, $\frac{3}{4}$ represents a quantity formed by 3 parts of size $\frac{1}{4}$.*

Cluster Headings	Content Standards
A. Develop understanding of fractions as numbers.	**3.NF.A.2** Understand a fraction as a number on the number line. Represent fractions on a number line. **a.** Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint locates the number $\frac{1}{b}$ on the number line. *For example, on a number line from 0 to 1, students can partition it into 4 equal parts and recognize that each part represents a length of $\frac{1}{4}$ and the first part has an endpoint at $\frac{1}{4}$ on the number line.* **b.** Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line. *For example, $\frac{5}{3}$ is the distance from 0 when there are 5 iterations of $\frac{1}{3}$.*
	3.NF.A.3 Explain equivalence of fractions and compare fractions by reasoning about their size. **a.** Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. **b.** Recognize and generate simple equivalent fractions (e.g., $\frac{1}{2} = \frac{2}{4}, \frac{4}{6} = \frac{2}{3}$) and explain why the fractions are equivalent using a visual fraction model. **c.** Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *For example, express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point on a number line diagram.* **d.** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	**3.MD.A.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve contextual problems involving addition and subtraction of time intervals in minutes. *For example, students may use a number line to determine the difference between the start time and the end time of lunch.* **3.MD.A.2** Measure the mass of objects and liquid volume using standard units of grams (g), kilograms (kg), milliliters (ml), and liters *(l)*. Estimate the mass of objects and liquid volume using benchmarks. *For example, a large paper clip is about one gram, so a box of about 100 large clips is about 100 grams. Therefore, ten boxes would be about 1 kilogram.*

Cluster Headings	Content Standards
B. Represent and interpret data.	**3.MD.B.3** Draw a scaled pictograph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled graphs. **3.MD.B.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units: whole numbers, halves, or quarters.
C. Geometric measurement: understand and apply concepts of area and relate area to multiplication and to addition.	**3.MD.C.5** Recognize that plane figures have an area and understand concepts of area measurement. **a.** Understand that a square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area. **b.** Understand that a plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **3.MD.C.6** Measure areas by counting unit squares (square centimeters, square meters, square inches, square feet, and improvised units). **3.MD.C.7** Relate area of rectangles to the operations of multiplication and addition. **a.** Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths. **b.** Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning. **c.** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b + c* is the sum of *a x b* and *a x c.* Use area models to represent the distributive property in mathematical reasoning. *For example, in a rectangle with dimensions 4 by 6, students can decompose the rectangle into 4 x 3 and 4 x 3 to find the total area of 4 x 6.* (See Table 3 - Properties of Operations) **d.** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.
D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	**3.MD.D.8** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry (G)

Cluster Headings	Content Standards
A. Reason about shapes and their attributes.	**3.G.A.1** Understand that shapes in different categories may share attributes and that the shared attributes can define a larger category. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories. **3.G.A.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area and describe the area of each part as 1/4 of the area of the shape.* **3.G.A.3** Determine if a figure is a polygon.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ **(1^{st})**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ **One-Step Problem** **(2^{nd})**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ **(1^{st})**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ **One-Step Problem** **(2^{nd})**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$ **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$ **(1^{st})**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1^{st})**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** **(1^{st})**	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $? + 3 = 5$ **One-Step Problem** **(2^{nd})**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$ **(1^{st})**	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$ **One-Step Problem** **(2^{nd})**	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** **(1^{st})**

K: Problem types to be mastered by the end of the Kindergarten year.

1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.

2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 2 Common multiplication and division situations[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Mathematics | Grade 4

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 4th grade.

Operations and Algebraic Thinking

Students build on their knowledge of multiplication and begin to interpret and represent multiplication as a comparison. They multiply and divide to solve contextual problems involving multiplicative situations, distinguishing their solutions from additive comparison situations. Students solve multi-step whole number contextual problems using the four operations representing the unknown as a variable within equations (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations). They apply appropriate methods to estimate and check for reasonableness. This is the first time students find and interpret remainders in context. Students find factors and multiples, and they identify prime and composite numbers. Students generate number or shape patterns following a given rule.

Number and Operations in Base Ten

Students generalize place value understanding to read and write numbers to 1,000,000, using standard form, word form, and expanded form. They compare the relative size of the numbers and round numbers to the nearest hundred thousand, which builds on 3rd grade rounding concepts. By the end of 4th grade, students should fluently add and subtract multi-digit whole numbers to 1,000,000. Students use strategies based on place value and the properties of operations to multiply a whole number up to four-digits by a one-digit number, and multiply two two-digit numbers. They use these strategies and the relationship between multiplication and division to find whole number quotients and remainders up to four-digit dividends and one-digit divisors (See Table 3 - Properties of Operations).

Number and Operations-Fractions

Students continue to develop an understanding of fraction equivalence by reasoning about the size of the fractions, using a benchmark fraction to compare the fractions, or finding a common denominator. Students extend previous understanding of unit fractions to compose and decompose fractions in different ways. They use the meaning of fractions and the meaning of multiplication as repeated addition to multiply a whole number by a fraction. Students solve contextual problems involving addition and subtraction of fractions with like denominators and multiplication of a whole number by a fraction (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions). Students learn decimal notation for the first time to represent fractions with denominators of 10 and 100. They express these fractions and their equivalents as decimals and are able to read, write, compare, and locate these decimals on a number line.

Measurement and Data

Students know the relative sizes of measurement units within one system of units and are able to convert within the single system of measurement. They use the four operations to solve contextual problems involving measurement. Students build on their previous understanding of area and perimeter to generate and apply formulas for finding the area and perimeter of rectangles. Students also build on their understanding of line plots and solve problems involving fractions using operations appropriate for the grade. For the first time, students learn concepts of angle measurement.

Geometry

Students extend their previous understanding to analyze and classify shapes based on line and angle types. Students also use knowledge of line and angle types to identify right triangles. Students recognize and draw lines of symmetry for the first time

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Use the four operations with whole numbers to solve problems. (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations)	**4.OA.A.1** Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.A.2** Multiply or divide to solve contextual problems involving multiplicative comparison, and distinguish multiplicative comparison from additive comparison. *For example, school A has 300 students and school B has 600 students: to say that school B has two times as many students is an example of multiplicative comparison; to say that school B has 300 more students is an example of additive comparison.* **4.OA.A.3** Solve multi-step contextual problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
B. Gain familiarity with factors and multiples.	**4.OA.B.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
C. Generate and analyze patterns.	**4.OA.C.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Generalize place value understanding for multi-digit whole numbers.	**4.NBT.A.1** Recognize that in a multi-digit whole number (less than or equal to 1,000,000), a digit in one place represents 10 times as much as it represents in the place to its right. *For example, recognize that 7 in 700 is 10 times bigger than the 7 in 70 because 700 ÷ 70 = 10 and 70 x 10 = 700.* **4.NBT.A.2** Read and write multi-digit whole numbers (less than or equal to 1,000,000) using standard form, word form, and expanded form (e.g. the expanded form of 4256 is written as 4 x 1000 + 2 x 100 + 5 x 10 + 6 x 1). Compare two multi-digit numbers based on meanings of the digits in each place and use the symbols >, =, and < to show the relationship. **4.NBT.A.3** Round multi-digit whole numbers to any place (up to and including the hundred-thousand place) using understanding of place value.

B. Use place value understanding and properties of operations to perform multi-digit arithmetic. (See Table 3 - Properties of Operations)	**4.NBT.B.4** Fluently add and subtract within 1,000,000 using appropriate strategies and algorithms. **4.NBT.B.5** Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations - Fractions (NF)

Limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Cluster Headings	Content Standards
A. Extend understanding of fraction equivalence and comparison.	**4.NF.A.1** Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{a \times n}{b \times n}$ or $\frac{a \div n}{b \div n}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. *For example,* $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$. **4.NF.A.2** Compare two fractions with different numerators and different denominators by creating common denominators or common numerators or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.

<table>
<tr><td>B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

(See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations for whole number situations that can be applied for fractions.)</td><td>4.NF.B.3 Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$. For example, $\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way (e.g., $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$), recording each decomposition by an equation. Justify decompositions by using a visual fraction model.
c. Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve contextual problems involving addition and subtraction of fractions referring to the same whole and having like denominators

4.NF.B.4 Apply and extend previous understandings of multiplication as repeated addition to multiply a whole number by a fraction.
a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times \frac{1}{4}$, recording the conclusion by the equation $\frac{5}{4} = 5 \times \frac{1}{4}$.
b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ and use this understanding to multiply a whole number by a fraction. For example, use a visual fraction model to express $3 \times \frac{2}{5}$ as $6 \times \frac{1}{5}$, recognizing this product as $\frac{6}{5}$.
(In general, $n \times \frac{a}{b} = \frac{(n \times a)}{b} = (n \times a) \times \frac{1}{b}$.)
c. Solve contextual problems involving multiplication of a whole number by a fraction (e.g., by using visual fraction models and equations to represent the problem). For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 4 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</td></tr>
<tr><td>C. Understand decimal notation for fractions and compare decimal fractions.</td><td>4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express, $\frac{3}{10}$ as $\frac{30}{100}$ and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.

4.NF.C.6 Read and write decimal notation for fractions with denominators 10 or 100. Locate these decimals on a number line.

4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.</td></tr>
</table>

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Estimate and solve problems involving measurement.	**4.MD.A.1** Measure and estimate to determine relative sizes of measurement units within a single system of measurement involving length, liquid volume, and mass/weight of objects using customary and metric units. **4.MD.A.2** Solve one- or two-step real-world problems involving whole number measurements with all four operations within a single system of measurement including problems involving simple fractions. **4.MD.A.3** Know and apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
B. Represent and interpret data.	**4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
C. Geometric measurement: understand concepts of angle and measure angles.	**4.MD.C.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. **a.** Understand that an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. **b.** Understand that an angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees and represents a fractional portion of the circle. **4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **4.MD.C.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems *(e.g., by using an equation with a symbol for the unknown angle measure)*.

Geometry (G)

Cluster Headings	Content Standards
A. Draw and identify lines and angles and classify shapes by properties of their lines and angles.	**4.G.A.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse, straight, reflex), and perpendicular and parallel lines. Identify these in two-dimensional figures. **4.G.A.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles. **4.G.A.3** Recognize and draw lines of symmetry for two-dimensional figures.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ **(1^{st})**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ **One-Step Problem** **(2^{nd})**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ **(1^{st})**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ **One-Step Problem** **(2^{nd})**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$ **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$ **(1^{st})**
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1^{st})**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** **(1^{st})**	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $? + 3 = 5$ **One-Step Problem** **(2^{nd})**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$ **(1^{st})**	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$ **One-Step Problem** **(2^{nd})**	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** **(1^{st})**

K: Problem types to be mastered by the end of the Kindergarten year.

1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.

2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 2 Common multiplication and division situations[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a+b)+c=a+(b+c)$
Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Distributive property of multiplication over addition	$a\times(b+c)=a\times b+a\times c$

Mathematics | Grade 5

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 5th grade.

Operations and Algebraic Thinking

Students build on their understanding of patterns to generate two numerical patterns using given rules and identify relationships between the patterns. For the first time, students form ordered pairs and graph them on a coordinate plane. In addition, students write and evaluate numerical expressions using parentheses and/or brackets.

Number and Operations in Base Ten

Students generalize their understanding of place value to include decimals by reading, writing, comparing, and rounding numbers. They recognize that in a multi-digit number, the value of each digit has a relationship to the value of the same digit in another position. Students explain patterns in products when multiplying a number by a power of 10. Whole-number exponents are used to denote powers of 10 for the first time. By the end of 5th grade, students should fluently multiply multi-digit whole numbers (up to 4 digits by 3 digits).

Students build on their understanding of why division procedures work based on place value and the properties of operations to find whole number quotients and remainders (See Table 3 - Properties of Operations). They apply their understanding of models for decimals, decimal notation, and properties of operations to add, subtract, multiply, and divide decimals to hundredths. (Limit division problems so that either the dividend or the divisor is a whole number.) They develop fluency in these computations and make reasonable estimates of their results. Students finalize their understanding of multi-digit addition, subtraction, multiplication, and division with whole numbers.

Number and Operations in Fractions

Students apply their understanding of equivalent fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions and make reasonable estimates of them. For the first time, students develop an understanding of fractions as division problems. They use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Limit to dividing unit fractions by whole numbers or whole numbers by unit fractions.) Students reason about the size of products compared to the size of the factors. Students should solve a variety of problem types in order to make connections among contexts, equations, and strategies (See Table 1 - Addition and Subtraction Situations and Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions).

Measurement and Data

Students build on their understanding of area and recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-sized units of volume required to fill the space without gaps or overlaps. Students decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of cubes. Students build on their understanding of measurements to convert from larger units to smaller units within a single system of measurement and solve multistep problems involving these conversions. Students solve problems with data from line plots involving fractions using operations appropriate for the grade.

Geometry

Students plot points on the coordinate plane to solve real-world and mathematical problems. Students classify two-dimensional figures into categories based on their properties.

Operations and Algebraic Thinking (OA)

Cluster Headings	Content Standards
A. Write and interpret numerical expressions.	**5.OA.A.1** Use parentheses and/or brackets in numerical expressions and evaluate expressions having these symbols using the conventional order (Order of Operations). **5.OA.A.2** Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.*
B. Analyze patterns and relationships.	**5.OA.B.3** Generate two numerical patterns using two given rules. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences.* **a.** Identify relationships between corresponding terms in two numerical patterns. *For example, observe that the terms in one sequence are twice the corresponding terms in the other sequence.* **b.** Form ordered pairs consisting of corresponding terms from two numerical patterns and graph the ordered pairs on a coordinate plane.

Number and Operations in Base Ten (NBT)

Cluster Headings	Content Standards
A. Understand the place value system.	**5.NBT.A.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.A.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **5.NBT.A.3** Read and write decimals to thousandths using standard form, word form, and expanded form (e.g., the expanded form of 347.392 is written as 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000)). Compare two decimals to thousandths based on meanings of the digits in each place and use the symbols >, =, and < to show the relationship. **5.NBT.A.4** Round decimals to the nearest hundredth, tenth, or whole number using understanding of place value.

Cluster Headings	Content Standards
B. Perform operations with multi-digit whole numbers and with decimals to hundredths. (See Table 3 - Properties of Operations)	**5.NBT.B.5** Fluently multiply multi-digit whole numbers (up to three-digit by four-digit factors) using appropriate strategies and algorithms. **5.NBT.B.6** Find whole-number quotients and remainders of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations; assess the reasonableness of answers using estimation strategies. (Limit division problems so that either the dividend or the divisor is a whole number.)

Number and Operations - Fractions (NF)

Cluster Headings	Content Standards
A. Use equivalent fractions as a strategy to add and subtract fractions. (See Table 1 - Addition and Subtraction Situations for whole number situations that can be applied to fractions)	**5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$. *(In general* $\frac{a}{b}+\frac{c}{d}=\frac{(ad+bc)}{bd}$.*)* **5.NF.A.2** Solve contextual problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$, *by observing that* $\frac{3}{7}<\frac{1}{2}$.
B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (See Table 2 - Multiplication and Division Situations for whole number situations that	**5.NF.B.3** Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b}=a\div b$). *For example,* $\frac{3}{4}=3\div 4$ *so when 3 wholes are shared equally among 4 people, each person has a share of size* $\frac{3}{4}$. Solve contextual problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models or equations to represent the problem. *For example, if 8 people want to share 49 sheets of construction paper equally, how many sheets will each person receive? Between what two whole numbers does your answer lie?*

Cluster Headings	Content Standards
B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (See Table 2 - Multiplication and Division Situations for whole number situations that can be applied to fractions)	**5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number or a fraction by a fraction. **a.** Interpret the product $\frac{a}{b}$ x q as a x (q ÷ b) (partition the quantity q into b equal parts and then multiply by a). Interpret the product $\frac{a}{b}$ x q as (a x q) ÷ b (multiply a times the quantity q and then partition the product into b equal parts). *For example, use a visual fraction model or write a story context to show that* $\frac{2}{3}$ *x 6 can be interpreted as 2 x (6 ÷ 3) or (2 x 6) ÷ 3. Do the same with* $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.) **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.
	5.NF.B.5 Interpret multiplication as scaling (resizing). **a.** Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, know if the product will be greater than, less than, or equal to the factors.* **b.** Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product less than the given number; and relate the principle of fraction equivalence $\frac{a}{b} = \frac{(a \times n)}{(b \times n)}$ to the effect of multiplying $\frac{a}{b}$ by 1.
	5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models or equations to represent the problem.
	5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. **a.** Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, use visual models and the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.* **b.** Interpret division of a whole number by a unit fraction and compute such quotients. *For example, use visual models and the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.* **c.** Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3 cup servings are in 2 cups of raisins?*

Measurement and Data (MD)

Cluster Headings	Content Standards
A. Convert like measurement units within a given measurement system from a larger unit to a smaller unit.	**5.MD.A.1** Convert customary and metric measurement units within a single system by expressing measurements of a larger unit in terms of a smaller unit. Use these conversions to solve multi-step real-world problems involving distances, intervals of time, liquid volumes, masses of objects, and money (including problems involving simple fractions or decimals). *For example, 3.6 liters and 4.1 liters can be combined as 7.7 liters or 7700 milliliters*
B. Represent and interpret data.	**5.MD.B.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
C. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	**5.MD.C.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **a.** Understand that a cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume and can be used to measure volume. **b.** Understand that a solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
	5.MD.C.4 Measure volume by counting unit cubes, using cubic centimeters, cubic inches, cubic feet, and improvised units.
	5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume of right rectangular prisms. **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent whole-number products of three factors as volumes *(e.g., to represent the associative property of multiplication).* **b.** Know and apply the formulas $V = l \times w \times h$ and $V = B \times h$ (where B represents the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Geometry (G)

Cluster Headings	Content Standards
A. Graph points on the coordinate plane to solve real-world and mathematical problems.	**5.G.A.1** Graph ordered pairs and label points using the first quadrant of the coordinate plane. Understand in the ordered pair that the first number indicates the horizontal distance traveled along the x-axis from the origin and the second number indicates the vertical distance traveled along the y-axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). **5.G.A.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.
B. Classify two-dimensional figures into categories based on their properties.	**5.G.B.3** Classify two-dimensional figures in a hierarchy based on properties. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**

Table 1 Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ (K)	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ (1st)	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ **One-Step Problem** (2nd)
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ (K)	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ (1st)	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ **One-Step Problem** (2nd)
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ (K)	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$ (K)	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$ (1st)
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (1st)	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **One-Step Problem** (1st)	(Version with "more"): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $? + 3 = 5$ **One-Step Problem** (2nd)
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$ (1st)	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$ **One-Step Problem** (2nd)	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **One-Step Problem** (1st)

K: Problem types to be mastered by the end of the Kindergarten year.

1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types.

2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

Table 2 Common multiplication and division situations[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Mathematics | Grade 6

The descriptions below provide an overview of the mathematical concepts and skills that students explore throughout the 6th grade.

Ratios and Proportional Relationships

6th grade begins the formal study of ratios and proportions. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates. Proportional relationships are added and studied in the 7th grade.

The Number System

Students use fractions, multiplication, and division along with an understanding of the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students also extend their previous understandings of numbers and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

Expressions and Equations

Students begin to use properties of arithmetic operations systematically to work with numerical expressions that contain whole-number exponents. Students come to understand more fully the use of variables and variable expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students explore how algebraic expressions can represent written situations and generalize relationships from specific cases.

Geometry

Students build on their work with area from earlier grades by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can more easily determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in the 7th grade by drawing polygons in the coordinate plane.

Statistics and Probability

6th grade students begin to formally develop their ability to think statistically. They understand that a set of data (collected to answer a question) will have a distribution, which can be described by its center, spread, and shape. Students calculate the median, mean, and mode and relate these to the overall shape of the distribution. They recognize that the median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. They understand that the mode refers to the most frequently occurring number found in a set of numbers and is found by collecting and organizing the data in order to count the frequency of each result. Students display, summarize and describe numerical data sets, considering the context in which the data were collected. Students use number lines, dot plots, box plots, and pie charts to display numerical data

Ratios and Proportional Relationships (RP)

Cluster Headings	Content Standards
A. Understand ratio concepts and use ratio reasoning to solve problems.	**6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, the ratio of wings to beaks in a bird house at the zoo was 2:1, because for every 2 wings there was 1 beak. Another example could be for every vote candidate A received, candidate C received nearly three votes*
	6.RP.A.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*. Use rate language in the context of a ratio relationship. *For example, this recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar. Also, we paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.* (Expectations for unit rates in 6^{th} grade are limited to non-complex fractions).
	6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations). **a.** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if a runner ran 10 miles in 90 minutes, running at that speed, how long will it take him to run 6 miles? How fast is he running in miles per hour?* **c.** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **d.** Use ratio reasoning to convert customary and metric measurement units (within the same system); manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System (NS)

Cluster Headings	Content Standards
A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	**6.NS.A.1** Interpret and compute quotients of fractions, and solve contextual problems involving division of fractions by fractions (e.g., using visual fraction models and equations to represent the problem is suggested). *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 times 8/9 is 2/3 ((*a/b*) ÷ (*c/d*) = ad/bc.)* *Further example: How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Cluster Headings	Content Standards
B. Compute fluently with multi-digit numbers and find common factors and multiples.	**6.NS.B.2** Fluently divide multi-digit numbers using a standard algorithm. **6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation. **6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
C. Apply and extend previous understandings of numbers to the system of rational numbers.	**6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
	6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself. *For example, – (–3) = 3, and that 0 is its own opposite.* **b.** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **c.** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6.NS.C.7 Understand ordering and absolute value of rational numbers. **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.* **c.** Understand the absolute value of a rational number as its distance from 0 on the number line and distinguish comparisons of absolute value from statements about order in a real-world context. *For example, an account balance of -24 dollars represents a greater debt than an account balance -14 dollars because -24 is located to the left of -14 on the number line*

Cluster Headings	Content Standards
C. Apply and extend previous understandings of numbers to the system of rational numbers.	**6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations (EE)

Cluster Headings	Content Standards
A. Apply and extend previous understandings of arithmetic to algebraic expressions.	**6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents.
	6.EE.A.2 Write, read, and evaluate expressions in which variables stand for numbers. **a.** Write expressions that record operations with numbers and with variables. *For example, express the calculation "Subtract y from 5" as 5 - y.* **b.** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* **c.** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
	6.EE.A.3 Apply the properties of operations (including, but not limited to, commutative, associative, and distributive properties) to generate equivalent expressions. The distributive property is prominent here. *For example, apply the distributive property to the expression* 3 (2 + x) *to produce the equivalent expression* 6 + 3x*; apply the distributive property to the expression* 24x + 18y *to produce the equivalent expression* 6 (4x + 3y)*; apply properties of operations to* y + y + y *to produce the equivalent expression* 3y.
	6.EE.A.4 Identify when expressions are equivalent (i.e., when the expressions name the same number regardless of which value is substituted into them). *For example, the expression* 5b + 3b *is equivalent to* (5 +3) b*, which is equivalent to* 8b.
B. Reason about and solve one-variable equations and inequalities.	**6.EE.B.5** Understand solving an equation or inequality is carried out by determining if any of the values from a given set make the equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Cluster Headings	Content Standards
B. Reason about and solve one-variable equations and inequalities.	**6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.B.7** Solve real-world and mathematical problems by writing and solving one-step equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. **6.EE.B.8** Interpret and write an inequality of the form $x > c$ or $x < c$ which represents a condition or constraint in a real-world or mathematical problem. Recognize that inequalities have infinitely many solutions; represent solutions of inequalities on number line diagrams.
C. Represent and analyze quantitative relationships between dependent and independent variables.	**6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another. *For example, Susan is putting money in her savings account by depositing a set amount each week (50). Represent her savings account balance with respect to the number of weekly deposits (*s = 50w*, illustrating the relationship between balance amount* s *and number of weeks* w*).* **a.** Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. **b.** Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Geometry (G)

Cluster Headings	Content Standards
A. Solve real-world and mathematical problems involving area, surface area, and volume.	**6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; know and apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Know and apply the formulas $V = lwh$ and $V = Bh$ *where B is the area of the base* to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Cluster Headings	Content Standards
A. Solve real-world and mathematical problems involving area, surface area, and volume.	**6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side that joins two vertices (vertical or horizontal segments only). Know and apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability (SP)

Cluster Headings	Content Standards
A. Develop understanding of statistical variability.	**6.SP.A.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (mean, median, mode), spread (range), and overall shape. **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
B. Summarize and describe distributions.	**6.SP.B.4** Display a single set of numerical data using dot plots (line plots), box plots, pie charts and stem plots.
	6.SP.B.5 Summarize numerical data sets in relation to their context. **a.** Report the number of observations. **b.** Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. **c.** Give quantitative measures of center (median and/or mean) and variability (range) as well as describing any overall pattern with reference to the context in which the data were gathered. **d.** Relate the choice of measures of center to the shape of the data distribution and the context in which the data were gathered.

Major content of the grade is indicated by the light green shading of the cluster heading and standard's coding.

	Major Content		**Supporting Content**